Upper Fox-Wolf Basins TMDL: WiLMS Lake Model Setup and Results

1 Introduction

This report outlines the setup and results of Wisconsin Lake Modeling Suite (WiLMS) water quality models for 21 lakes in the Upper Fox and Wolf Basins (UFWB). The 21 lakes included in WiLMS modeling (Table 1) are present on Wisconsin's draft 2016 Impaired Waters List and require Total Maximum Daily Loads (TMDLs) to address issues of nutrient enrichment.

The WiLMS water quality models were developed by The Cadmus Group, Inc. to support TMDL development efforts by the US Environmental Protection Agency (EPA) Region 5 and the Wisconsin Department of Natural Resources (WDNR). The WiLMS models use information on lake morphology, water inflows, and phosphorus loading to provide estimates of in-lake phosphorus concentrations to guide TMDL analysis.

Table 1. UFWB lakes included in WiLMS lake water quality modeling.

Lake Name	WBIC	County
Big Twin Lake	146500	Green Lake
Black Otter Lake	315600	Outagamie
Buffalo Lake	168000	Marquette
Collins Lake	270200	Portage
Crane Lake	388500	Forest
Green Lake	146100	Green Lake
Lake Emily	161600	Dodge
Little Green Lake	162500	Green Lake
Long Lake	321300	Shawano
Mason Lake	175700	Adams, Marquette
Old Taylor Lake	195000	Waupaca
Park Lake	180300	Columbia
Pine Lake	406900	Forest
Puckaway Lake	158700	Marquette, Green Lake
School Section Lake	283600	Waupaca
Shawano Lake	322800	Shawano
Spring Lake	267200	Portage
Swan Lake	179800	Columbia
Tree Lake	289400	Portage
Upper Post Lake	399200	Langlade, Oneida
White Clay Lake	326400	Shawano

2 WiLMS Description

WiLMS is a lake water quality modeling tool developed by WDNR. Key conceptual features of WiLMS are described in the WiLMS user manual (WDNR 2003) and summarized below:

- A lake is represented as a zero-dimensional, completely-mixed body of water with no horizontal or vertical variability in water quality;
- Water quality is modeled on an annual time step. Lake total phosphorus (TP) concentrations predicted by WiLMS are growing season averages for the year being modeled;
- Lake TP concentrations can be predicted using one of several empirical equations. The empirical equations used in WiLMS were derived from statistical analysis of field data from multiple lakes across the US.

Early versions of WiLMS were released as a Microsoft Excel workbook pre-programmed with formulas for predicting lake water quality. WiLMS has since been released as standalone software program with a graphical user interface for entering inputs and viewing outputs. Because the current effort involved setting up many models, the Excel version of WiLMS (version 2.01) was used. Formulas in the Excel version were updated to ensure that outputs were consistent with the most recent version of WiLMS available at the time of this report (version 3.3).

3 Model Setup

The WiLMS modeling effort consisted of applying the following steps for each of the 21 lakes listed in Table 1:

1. Compile water quality monitoring data for the years 2000 through 2013 and identify years in which total phosphorus concentrations were sampled in the lake. Divide the sample data period of record into a calibration period and a validation period.
2. Setup a WiLMS model file for the calibration period that simulates average conditions during calibration years;
3. Calibrate WiLMS output by calculating a calibration factor as the ratio of observed to predicted TP concentrations during the calibration period;
4. Setup a WiLMS model file for the validation period that simulates average conditions during validation years;
5. Evaluate model predictions by applying the adjustment factor calculated in step 3 to the predicted TP concentration for the validation period and comparing to the observed TP concentration.

This section describes the sources of input data used to setup WiLMS models. Required inputs include lake morphology (surface area and volume), annual water inflow, and annual total phosphorus loading to the lake. Also described in this section are the observed total phosphorus concentration data used for model calibration and validation.

3.1 Lake Morphology

Lake surface area and volume for 17 of the 21 lakes were set to values reported on WDNR lake survey maps (http://dnr.wi.gov/lakes/maps/). Survey maps for Lake Emily, Pine Lake, and School Section Lake did not report surface area and volume. For these lakes, survey maps were digitized using

Geographic Information System (GIS) software and surface area and volume were calculated using the digitized bathymetry. A survey map was not available for Black Otter Lake. The surface area of Black Otter Lake was set to the value reported on the WDNR lake information webpage (http://dnr.wi.gov/lakes/ lakepages/LakeDetail.aspx?wbic =315600). The volume of Black Otter Lake was set to the value reported in the *Black Otter Lake Adaptive Lake Management Plan* (STS, 2008). Lake morphology parameter values are listed for each lake in Table 2.

3.2 Water Inflow

Annual water inflow to each lake (Table 3) was estimated using the SWAT model developed for the UFWB TMDL (The Cadmus Group, 2015). Of the 21 lakes included in WiLMS modeling, 16 lakes are located at the outlet of a SWAT subwatershed (i.e., the lake watershed boundary matches the SWAT subwatershed boundary; see Figure 1, left). For these lakes, annual flow rates from the SWAT reach output file were extracted and used as estimates of annual water inflow to the lake.

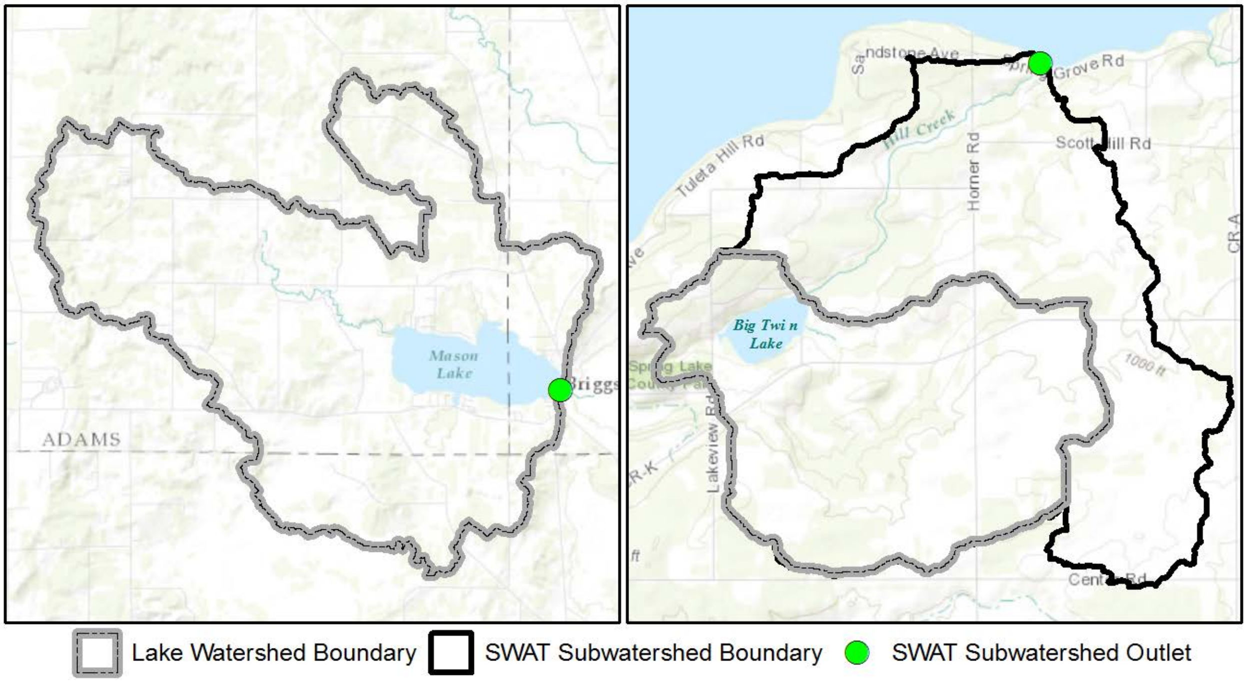

Figure 1.Examples of the location of modeled lakes within SWAT subwatersheds. In the example on the left, the lake is located at the SWAT subwatershed outlet and the lake watershed matches the SWAT subwatershed. In the example on the right, the lake is located upstream of the SWAT subwatershed outlet and the SWAT subwatershed extends beyond the lake watershed.

Two alternative approaches were used for deriving annual water inflows to Big Twin Lake, Lake Emily, Old Taylor Lake, Shawano Lake, and Spring Lake because these lakes are located upstream of a SWAT subwatershed outlet (i.e., the SWAT subwatershed boundary extends beyond the lake watershed boundary; see Figure 1, right).

Shawano Lake is located near the outlet of SWAT subwatershed 41. Annual water inflows to Shawano Lake were estimated by extracting annual flow rates for subwatershed 41 from the SWAT reach output file and multiplying by an area adjustment factor of 0.94. The area adjustment factor was calculated as the ratio of the Shawano Lake watershed area to the drainage area of SWAT subwatershed 41. This method assumes that 94% of the annual flow from SWAT subwatershed 41 comes from the Shawano Lake watershed.

Big Twin Lake, Lake Emily, Old Taylor Lake, and Spring Lake are located in the upper reaches of SWAT subwatersheds. Annual lake inflows were estimated for these lakes using SWAT predictions of runoff from Hydrologic Response Units (HRUs). HRUs are unique land use-soil-slope combinations within a subwatershed and are the fundamental land units of SWAT water balance calculations. Annual lake inflows were estimated by first extracting annual runoff rates from the SWAT HRU output file for all HRUs in the SWAT subwatersheds where Big Twin Lake, Lake Emily, Old Taylor Lake, and Spring Lake are located. For each subwatershed, annual HRU runoff volumes were then summed by major land use category and multiplied by an area adjustment factor. Area adjustment factors were calculated as the ratio of land use area in the lake watershed to land use area in the SWAT subwatershed. Separate adjustment factors were calculated for all lake-land use combinations. Area-adjusted runoff volumes calculated for each land use category in the lake watershed were then summed to estimate total annual flow into the lake.

An additional source of water inflow to a lake is direct precipitation onto the water surface. WiLMS allows users to input net precipitation onto the lake surface (net precipitation is total precipitation minus evaporation). Because evaporation data were not available for the modeled lakes, net precipitation was set to the default value stored in WiLMS for the county that each lake is located in (Table 2).

3.3 Phosphorus Loading

Annual phosphorus loading to each lake (Table 4) was also estimated using the SWAT model developed for the UFWB TMDL (The Cadmus Group, 2015). Methods for deriving annual phosphorus loads from SWAT output followed methods used for estimating water inflows. For the 16 lakes with a SWAT subwatershed outlet point at the lake outlet, annual phosphorus loads from the SWAT reach output file were extracted and used as annual phosphorus loads into the lake.

For Shawano Lake, annual phosphorus loads for SWAT subwatershed 41 were extracted from the SWAT reach output file. Loads were then multiplied by an area adjustment factor of 0.94, based on the ratio of the Shawano Lake watershed area to the SWAT subwatershed area.

For Big Twin Lake, Lake Emily, Old Taylor Lake, and Spring Lake, annual phosphorus loads were estimated by extracting annual phosphorus loads from the SWAT HRU output file for all HRUs in the SWAT subwatersheds that the four lakes are located in. Annual HRU loads were then summed by major land use category and multiplied by area adjustment factors. Area adjustment factors were calculated as the ratio of land use area in the lake watershed to land use area in the SWAT subwatershed. Separate adjustment factors were calculated for all lake-land use combinations. Area-adjusted loads for each land use category in the lake watershed were then summed to estimate annual phosphorus loading into the lake.

The UFWB SWAT model does not explicitly simulate phosphorus loading to lakes from onsite wastewater treatment (septic) systems. Phosphorus loading from nearshore septic systems was therefore included as an additional source of phosphorus in each lake model. WiLMS includes the following equation for calculating phosphorus loading from septic systems:

$$L = E * P * (1 - R)$$

L = Annual phosphorus load from septic systems (kilograms/year)
E = Septic tank phosphorus export rate (kilograms/person/year)
C = Population using septic systems (persons)
R = Phosphorus retention coefficient (dimensionless)

In all WiLMS models, the phosphorus export rate from septic systems (*E*) was to the default WiLMS value of 0.8 kilograms/person/year.

The population using septic systems (*C*) was estimated for each lake using a count of homes that are within 500 feet of the lake shore and outside of municipal sanitary sewer service areas. Initial counts of homes with septic systems were derived using GIS map layers of land parcel boundaries, sanitary sewer system boundaries, and aerial photos. Initial counts were refined using input from county land planning department staff. Counts of homes with septic systems were multiplied by the county average number of persons per household from the 2010 US Census to convert to the number of persons using septic systems and multiplied by 0.75 to account for non-permanent residents.

The phosphorus retention coefficient (*R)* describes how much of the phosphorus that is exported from a septic system enters the lake. A value of 0 means that all of the exported phosphorus enters the lake, while a value of 1 means that none of the exported phosphorus enters the lake. Phosphorus retention depends on several factors, including the type and condition of the septic system, the properties of soils that the septic system drains to, the distance from the septic system to the lake, and the direction of groundwater flow in the vicinity of the septic system. The WiLMS user manual (WDNR 2003) states that phosphorus retention coefficients range from 0.80 to 0.98 for properly functioning septic systems.

Phosphorus retention coefficients for each lake were assigned based on a review of the properties of soils in the 500 foot buffer surrounding the lake. Retention coefficient values were selected based on soil texture and soil hydrologic group listed in the US Department of Agriculture Natural Resource Conservation Service (USDA NRCS) Soil Survey Geographic (SSURGO) database for nearshore soils. Lakes with very high infiltration rates were assumed to have a lower phosphorus retention coefficient because they could allow rapid drainage of septic effluent into the lake. Soils with very low infiltration rates were also assumed to have a lower phosphorus retention coefficient because septic effluent could pond at the land surface and runoff into the lake during wet weather conditions. Lakes with predominantly sandy nearshore soils and high infiltration rates (hydrologic group A) were assigned a phosphorus retention coefficient of 0.8. Lakes with predominantly clayey nearshore soils and very low infiltration rates (hydrologic group C) were assigned a phosphorus retention coefficient of 0.8. Lakes with predominantly loamy nearshore soils and moderate to low infiltration rates (hydrologic groups B or C) were assigned a phosphorus retention coefficient of 0.9. Lakes with a mix of soils with varied textures and hydrologic groups were assigned a phosphorus retention coefficient of 0.85.

Septic system parameter values for each lake are listed in Table 2.

Table 2. Lake morphology, net precipitation, and septic system parameter values used in WiLMS lake water quality models.

Lake Name	WBIC	Watershed Area (acres)	Surface Area (acres)	Volume (acre-feet)	Mean Depth (feet)	Net Precipitation (inches/year)	Septic Population (persons)	Septic P Retention
Big Twin Lake	146500	2,108	78	1,286	16	3.1	40.6	0.9
Black Otter Lake	315600	10,021	75	384	5	3.4	0	-
Buffalo Lake	168000	253,126	2,210	10,180	5	3	850.2	0.8
Collins Lake	270200	1,829	42	1,036	25	3.9	35.7	0.9
Crane Lake	388500	3,818	337	3,922	12	5.3	181.2	0.9
Green Lake	146100	65,871	7	761	104	3.1	352.8	0.9
Lake Emily	161600	1,562	268	1,414	5	2.4	99.9	0.9
Little Green Lake	162500	2,391	466	4,817	10	3.1	458.7	0.9
Long Lake	321300	6,080	86	1,601	19	4.6	5.4	0.8
Mason Lake	175700	18,897	856	5,784	7	2.8	284.6	0.85
Old Taylor Lake	195000	172	55	265	5	3.8	64	0.9
Park Lake	180300	34,139	312	2,187	7	2.4	296.1	0.9
Pine Lake	406900	13,075	1,500	16,976	11	5.3	462.9	0.9
Puckaway Lake	158700	495,816	5,037	15,327	3	3.05	631.6	0.8
School Section Lake	283600	1,644	36	678	19	3.8	12.5	0.9
Shawano Lake	322800	43,375	6,063	54,270	9	4.6	5.4	0.85
Spring Lake	267200	7,052	37	311	8	3.9	10.7	0.8
Swan Lake	179800	41,989	406	12,898	32	2.4	263.4	0.8
Tree Lake	289400	2,951	74	1,051	14	3.9	84	0.9
Upper Post Lake	399200	64,176	757	4,782	6	5.6	656.2	0.85
White Clay Lake	326400	2,701	234	3,166	14	4.6	10.7	0.85

Table 3. Estimated annual water inflow (in inches per year) compiled for WiLMS lake water quality modeling.

Lake Name	**WBIC**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Big Twin Lake	146500	5.0	7.8	6.3	4.0	13.5	4.6	3.7	6.0	12.4	5.3	5.5	5.8	6.6	5.1
Black Otter Lake	315600	6.3	8.7	10.4	9.7	12.6	8.1	7.5	6.5	10.5	8.3	15.9	15.8	8.0	10.5
Buffalo Lake	168000	9.4	11.3	8.9	6.9	14.1	5.9	8.5	8.1	17.3	11.1	12.7	12.7	10.4	12.3
Collins Lake	270200	11.9	12.1	14.0	11.5	13.2	10.5	12.9	11.2	11.8	12.7	16.1	16.1	12.9	15.4
Crane Lake	388500	12.3	12.1	16.8	12.9	14.2	11.0	11.7	11.9	11.6	9.4	12.5	12.2	12.3	14.2
Green Lake	146100	6.0	9.0	7.3	4.7	14.9	5.6	4.7	6.7	14.1	6.7	6.5	6.8	7.9	6.3
Lake Emily	161600	7.1	8.6	6.8	5.0	11.0	4.2	5.7	5.7	13.5	7.5	9.1	8.7	7.0	8.0
Little Green Lake	162500	8.2	11.2	9.7	7.3	17.5	7.9	6.9	9.3	16.1	8.2	8.7	9.1	9.8	7.8
Long Lake	321300	5.8	5.1	5.9	5.8	7.9	5.8	6.8	5.1	6.8	5.5	9.1	11.1	6.2	9.9
Mason Lake	175700	10.6	13.2	9.3	8.5	16.7	5.9	11.0	9.6	19.9	11.5	14.4	13.9	10.9	14.1
Old Taylor Lake	195000	7.4	8.7	8.0	7.8	10.8	7.2	6.6	8.5	9.6	8.7	11.7	12.8	9.7	9.4
Park Lake	180300	10.9	13.1	10.6	7.4	16.4	6.7	8.8	8.6	20.3	12.0	13.9	13.8	11.0	12.9
Pine Lake	406900	10.5	10.5	15.4	11.2	11.7	9.9	12.3	11.1	11.4	9.3	10.2	13.6	7.9	8.8
Puckaway Lake	158700	9.0	11.1	8.8	6.7	14.1	5.9	8.1	8.0	17.0	10.9	12.3	12.3	10.3	11.8
School Section Lake	283600	8.9	8.7	9.4	8.0	9.2	6.0	6.3	4.3	9.5	7.4	11.9	9.6	6.4	9.3
Shawano Lake	322800	8.4	8.3	8.7	8.2	10.7	8.3	9.2	7.5	9.1	7.8	11.2	13.4	8.9	11.8
Spring Lake	267200	7.1	8.5	7.3	7.2	10.3	6.5	6.3	7.8	8.9	8.1	11.8	12.3	8.3	9.1
Swan Lake	179800	10.6	12.7	10.4	7.1	16.0	6.5	8.5	8.4	19.7	11.8	13.5	13.5	10.7	12.6
Tree Lake	289400	9.1	9.3	11.6	9.4	12.3	9.0	12.4	10.1	11.4	12.0	15.9	16.1	11.1	14.7
Upper Post Lake	399200	8.5	8.2	13.2	9.3	9.8	8.1	10.6	9.3	9.9	7.8	8.4	11.7	6.1	7.0
White Clay Lake	326400	11.2	9.4	9.4	9.3	11.5	9.3	10.8	8.0	10.8	9.3	14.4	14.7	9.1	14.0

Table 4. Estimated annual total phosphorus loading (in kilograms per hectare per year) compiled for WiLMS lake water quality modeling. Note that phosphorus loading from nearshore septic systems was included as an additional source of phosphorus in WiLMS models using septic parameters displayed in Table 2.

Lake Name	**WBIC**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
Big Twin Lake	146500	0.35	0.55	0.45	0.22	1.32	0.20	0.20	0.52	1.18	0.25	0.33	0.35	0.77	0.39
Black Otter Lake	315600	0.56	0.62	1.14	0.59	0.92	0.42	0.65	0.37	1.52	0.53	1.07	1.50	0.61	1.10
Buffalo Lake	168000	0.24	0.24	0.19	0.11	0.43	0.09	0.15	0.13	0.48	0.21	0.25	0.28	0.25	0.31
Collins Lake	270200	0.08	0.07	0.09	0.07	0.09	0.06	0.08	0.06	0.08	0.08	0.11	0.10	0.08	0.11
Crane Lake	388500	0.08	0.08	0.14	0.08	0.10	0.09	0.07	0.07	0.09	0.05	0.08	0.08	0.08	0.09
Green Lake	146100	0.30	0.41	0.33	0.17	0.88	0.16	0.16	0.31	0.89	0.20	0.26	0.27	0.50	0.27
Lake Emily	161600	0.53	0.53	0.32	0.21	1.01	0.16	0.29	0.27	0.93	0.38	0.52	0.50	0.44	0.54
Little Green Lake	162500	0.18	0.30	0.23	0.12	0.80	0.13	0.10	0.27	0.61	0.14	0.19	0.20	0.40	0.18
Long Lake	321300	0.10	0.09	0.14	0.11	0.16	0.10	0.15	0.08	0.24	0.10	0.17	0.23	0.11	0.25
Mason Lake	175700	0.37	0.37	0.28	0.16	0.68	0.10	0.23	0.18	0.81	0.29	0.37	0.41	0.41	0.48
Old Taylor Lake	195000	0.07	0.07	0.08	0.06	0.09	0.06	0.06	0.09	0.10	0.07	0.09	0.13	0.09	0.13
Park Lake	180300	0.56	0.57	0.42	0.23	0.98	0.20	0.33	0.29	1.10	0.44	0.56	0.59	0.54	0.65
Pine Lake	406900	0.10	0.11	0.17	0.10	0.12	0.11	0.12	0.10	0.11	0.10	0.09	0.13	0.07	0.07
Puckaway Lake	158700	0.25	0.28	0.21	0.12	0.52	0.11	0.16	0.17	0.57	0.23	0.27	0.29	0.31	0.32
School Section Lake	283600	0.37	0.25	0.53	0.23	0.29	0.17	0.32	0.15	0.73	0.25	0.43	0.43	0.25	0.57
Shawano Lake	322800	0.10	0.10	0.13	0.10	0.15	0.10	0.13	0.09	0.20	0.10	0.16	0.21	0.12	0.21
Spring Lake	267200	0.19	0.16	0.20	0.14	0.19	0.10	0.16	0.18	0.35	0.16	0.23	0.30	0.22	0.34
Swan Lake	179800	0.50	0.50	0.37	0.20	0.86	0.18	0.29	0.26	0.98	0.39	0.49	0.53	0.47	0.57
Tree Lake	289400	0.09	0.08	0.10	0.08	0.10	0.07	0.13	0.09	0.13	0.09	0.14	0.16	0.11	0.19
Upper Post Lake	399200	0.11	0.10	0.20	0.10	0.12	0.13	0.14	0.11	0.12	0.09	0.10	0.14	0.07	0.08
White Clay Lake	326400	0.12	0.09	0.14	0.09	0.16	0.11	0.15	0.08	0.29	0.10	0.21	0.24	0.11	0.26

3.4 Observed Total Phosphorus Concentrations

TP concentration samples collected from the 21 modeled lakes from 2000 through 2013 were acquired from the WDNR Surface Water Integrated Monitoring System (SWIMS) database. Data cleaning steps were applied to remove samples from monitoring stations that were not regularly sampled over several years.

The period of record for each lake was defined as years with TP samples collected in at least two growing season months (June through September). The period of record was divided into a model calibration period and a model validation period, with more recently sampled years assigned to the calibration period. Growing season samples were then averaged for the calibration period and the validation period for comparison to WiLMS TP concentration predictions. The following steps were applied to calculate growing season mean TP concentrations:

- Calculate the daily mean TP concentration for days with multiple TP samples;
- Calculate the monthly mean TP concentration for months with multiple TP samples;
- Calculate the growing season mean TP concentration from monthly means.

Table 5 lists the calibration and validation years for each lake and calculated growing season mean TP concentrations. Note that seven lakes had a limited monitoring record (6 years or less) that was not sufficient for defining a validation period (Big Twin Lake, Black Otter Lake, Buffalo Lake, Lake Emily, Old Taylor Lake, School Section Lake, and Spring Lake). Calibrated WiLMS predictions were therefore not compared to sample data for these lakes.

Table 5. Calibration and validation periods for each lake and mean growing season total phosphorus (TP) concentrations.

Lake Name	WBIC	Calibration Period	Validation Period	Calibration Period Growing Season Mean TP (µg/L)	Validation Period Growing Season Mean TP (µg/L)
Big Twin Lake	146500	2004-2006; 2009	None	43	-
Black Otter Lake	315600	2002; 2009-2013	None	100	-
Buffalo Lake	168000	2000-2001	None	135	-
Collins Lake	270200	2009-2013	2001; 2003; 2005-2008	27	23
Crane Lake	388500	2009-2013	2002-2006	32	28
Green Lake	146100	2009-2013	2004-2008	17	22
Lake Emily	161600	2005-2006; 2011-2013	None	64	-
Little Green Lake	162500	2011-2013	2000-2005	96	137
Long Lake	321300	2008-2013	2001-2007	35	30
Mason Lake	175700	2008-2013	2000-2002; 2004; 2006; 2007	125	101
Old Taylor Lake	195000	2003-2005	None	49	-
Park Lake	180300	2009; 2011-2013	2004; 2006; 2007	106	112
Pine Lake	406900	2009-2013	2004-2008	37	28
Puckaway Lake	158700	2009; 2011-2013	2004-2007	142	139
School Section Lake	283600	2009-2013	None	35	-
Shawano Lake	322800	2007-2013	2000-2006	44	32
Spring Lake	267200	2012-2013	None	30	-
Swan Lake	179800	2006-2007; 2009-2013	2000-2005	30	29
Tree Lake	289400	2010-2013	2007-2009	24	24
Upper Post Lake	399200	2010-2013	2001; 2007-2009	46	49
White Clay Lake	326400	2010-2013	2005; 2006; 2008	39	42

3.5 Empirical Model Selection

WiLMS includes several empirical regression equations to predict the TP concentration of a lake under alternative phosphorus loading magnitudes. These are referred to as “lake response models” because they quantify the relationship between phosphorus loading and in-lake phosphorus concentrations. This study focused on predictions from the Canfield-Bachmann response models. The Canfield-Bachmann models were developed from a database of 723 natural lakes and reservoirs throughout the United States, Canada, and northern Europe. The Canfield-Bachmann models are described in detail in Canfield and Bachmann (1981).

Two separate Canfield-Bachmann models are available for TP prediction; one derived from empirical analysis of data from natural lakes and one derived from analysis of artificial lakes. The Canfield-Bachmann model for natural lakes is:

$$P = \frac{L}{z\left[\left(0.162\frac{L}{z}\right)^{0.458} + p\right]}$$

where *P* is lake growing season mean TP concentration, *L* is annual areal total phosphorus load into the lake, *z* is lake mean depth, and *p* is lake hydraulic flushing rate.

The Canfield-Bachmann model for artificial lakes is:

$$P = \frac{L}{z\left[\left(0.114\frac{L}{z}\right)^{0.589} + p\right]}$$

Canfield and Bachmann (1981) report the characteristics of natural and artificial lakes in the dataset used to develop the empirical models, including residence time, mean depth, total phosphorus concentration, and areal phosphorus loading (i.e., phosphorus load per unit of lake surface area) (Table 6). These characteristics were used to determine whether the natural or artificial equation should be applied to each of the 21 lakes in this study. Nineteen (19) of the 21 lakes in this study matched the profile of natural lakes in the Canfield-Bachmann dataset, while 2 lakes (Black Otter Lake and Park Lake) better fit the profile of artificial lakes. The Canfield-Bachmann model for natural lakes was therefore applied for all lakes except for Black Otter Lake and Park Lake, which used the artificial lake model.

Table 6. Characteristics of natural and artificial lakes in the Canfield-Bachmann dataset (Canfield and Bahmann, 1981).

Lake Characteristic	**Natural Lakes**	**Artificial Lakes**
Residence Time (years)	Mean = 0.2 Range = 0.005 to 1,000	Mean = 0.003 Range = 0.001 to 53
Mean Depth (feet)	Mean = 42 Range = 0.7 to 1,007	Mean = 30 Range = 2 to 194
Areal TP Load (pounds per acre)	Mean = 25 Range = 0.3 to 678	Mean = 134 Range = 0.4 to 7,316
TP Concentration (micrograms per liter)	Mean = 120 Range = 4 to 2,600	Mean = 78 Range = 6 to 1,500

4 Model Results

4.1 Model Calibration

WiLMS model files were initially setup to predict growing season mean TP concentrations during the calibration period for the 21 lakes in this study. A calibration factor was then applied to the predicted TP concentration based on a comparison to the observed TP concentration during the calibration period. This approach was used as a form of model calibration to account the difference between each lake's individual loading response relationship and the generalized relationship represented by the Canfield-Bachmann model. The calibration factor was calculated for each lake as the ratio of the observed to predicted growing season mean TP concentration. Calibration factors are listed in Table 7.

Table 7. Comparison of observed and predicted growing season mean TP concentrations for the calibration period. Also displayed are calibration factors for each lake, calculated as the ratio of the observed to predicted TP concentration.

Lake Name	**Observed TP (µg/L)**	**Predicted TP (µg/L)**	**Adjustment Factor**
Big Twin Lake	43	88	0.49
Black Otter Lake	100	175	0.57
Buffalo Lake	135	74	1.83
Collins Lake	27	19	1.47
Crane Lake	32	16	1.99
Green Lake	17	18	0.98
Lake Emily	64	65	0.98
Little Green Lake	96	36	2.66
Long Lake	35	50	0.71
Mason Lake	125	75	1.68
Old Taylor Lake	49	23	2.07
Park Lake	106	99	1.06
Pine Lake	37	18	2.07
Puckaway Lake	142	83	1.70
School Section Lake	35	77	0.46
Shawano Lake	44	26	1.67
Spring Lake	30	96	0.31
Swan Lake	30	80	0.37
Tree Lake	24	29	0.82
Upper Post Lake	46	38	1.22
White Clay Lake	39	28	1.37

4.2 Model Validation

After calculating calibration factors, an additional set of WiLMS model files were setup to predict the growing season mean TP concentration during the validation period for each lake with validation data (note that Big Twin Lake, Black Otter Lake, Buffalo Lake, Lake Emily, Old Taylor Lake, School Section Lake, and Spring Lake did not have sufficient monitoring data for validation). The accuracy of TP predictions was then evaluated by:

1. Multiplying the predicted growing season mean TP concentration for the validation period by the lake's calibration factor listed in Table 7; and

2. Comparing the adjusted TP concentration calculated in step 1 to the observed growing season mean TP concentration for the validation period.

Table 8 lists observed and predicted growing season mean TP concentrations for the validation period. Predicted TP concentrations for eleven of the fourteen lakes with validation data are within ±15% of observed TP concentrations. Lakes with a greater difference between predicted and observed TP concentrations are Crane Lake (predicted concentration is 29% higher than observed concentration), Little Green Lake (predicted concentration is 28% lower than observed concentration), and Pine Lake (predicted concentration is 40% higher than observed concentration). The differences between predicted and observed TP concentrations are likely due to a combination of:

- Input data – Errors in estimates of precipitation, evaporation, water inflow, and nutrient loading can result in inaccurate predictions of lake TP concentrations; and
- Monitoring data –The frequency of water quality sampling may not be sufficient for accurately characterizing average conditions.

Given the uncertainty associated with input and monitoring data, the differences between predicted and observed TP concentrations during the validation period are within an acceptable range for further application of the calibrated WiLMS models for phosphorus TMDL development.

Table 8. Comparison of observed and predicted growing season mean TP concentrations for the validation period. Note that predicted TP concentrations are adjusted by the calibration factors listed in Table 7.

Lake Name	**Observed TP (µg/L)**	**Predicted TP (µg/L)**	**% Difference**
Collins Lake	23	24	3%
Crane Lake	28	36	29%
Green Lake	22	22	-2%
Little Green Lake	137	98	-28%
Long Lake	30	30	1%
Mason Lake	101	112	11%
Park Lake	112	110	-2%
Pine Lake	28	40	40%
Puckaway Lake	139	144	4%
Shawano Lake	32	37	15%
Swan Lake	29	32	7%
Tree Lake	24	20	-15%
Upper Post Lake	49	48	-3%
White Clay Lake	42	40	-6%

5 Conclusions

Wisconsin Lake Modeling Suite (WiLMS) lake water quality models were developed for 21 lakes in the Upper Fox and Wolf Basins (UFWB). The WiLMS models are empirical lake response models that predict the growing season mean total phosphorus (TP) concentration in a lake given its annual TP load. The WiLMS models were calibrated by comparing observed and predicted TP concentrations during a calibration period and calculating a calibration factor as the ratio of the observed to predicted TP concentration. The accuracy of the calibrated model predictions was evaluated by comparing observed and predicted TP concentrations during a separate validation period. Eleven of the fourteen lakes with validation data had predicted TP concentrations within ±15% of observed concentrations and predicted TP concentrations were within ±30-40% of observed concentrations for the remaining three lakes. The calibrated WiLMS models will be used for TMDL development by determining the phosphorus load that corresponds to attainment of numeric phosphorus criteria for each of the 21 lakes.

6 References

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