QUANTITATIVE EXERCISES USING THE GROUNDWATER SAND TANK MODEL

Darcy's Law

In 1856 Henri Darcy, a French engineer, discovered a relationship that governs fluid flow through geologic materials. He determined that flow through a sand-filled pipe was proportional to the change in head (the height to which the water rises in a small tube) divided by the length of the pipe (dh/dl). K (the proportionality constant) is known as the hydraulic conductivity and is a measure of the capacity for a porous medium to transmit water. It has high values for sand and lower values for rock and clay. The relationship of discharge to head, known as Darcy's Law, can be written as:

Q= - KiA

Where: Q = the volume of water discharged in a given time period. (length cubed/time; e. g. ft^3/day)
K = the hydraulic conductivity of the aquifer matrix. (length/time) The negative sign is to show flow from higher head to lower head.
i = the gradient of the water table or piezometric surface (dh/dl or rise over run or slope) (unitless)
A = the cross-sectional area perpendicular to the flow direction (length x length)

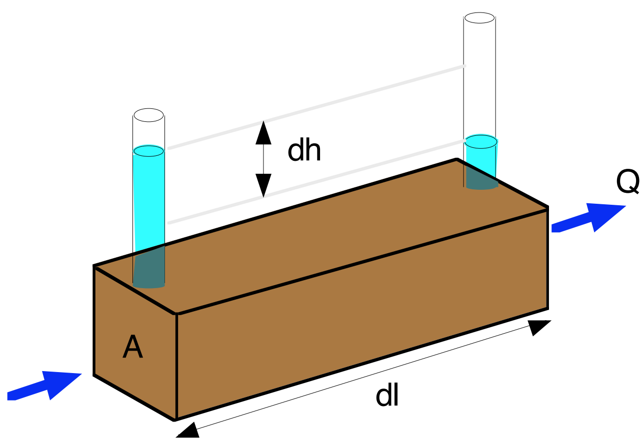
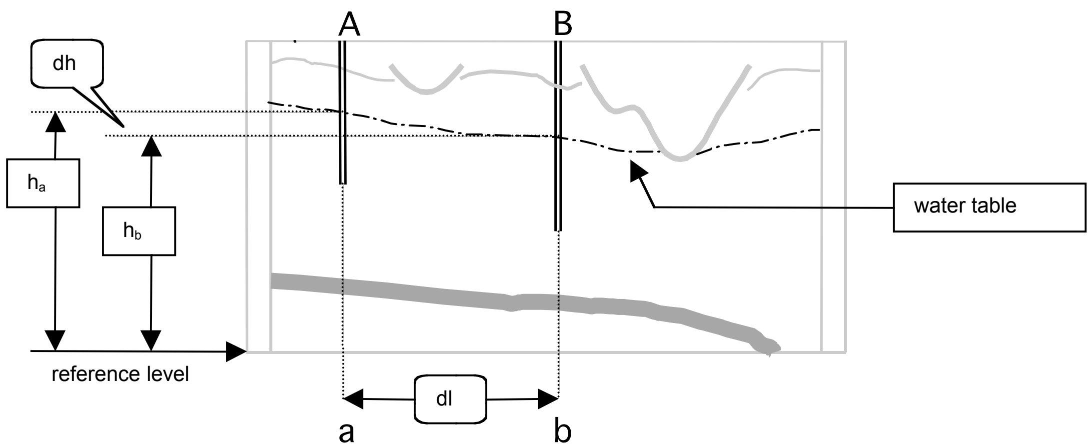


Figure illustrating Darcy's law.

With your groundwater model you can measure the hydraulic conductivity of the material in the model and then make and check predictions of travel times in the modeled aquifer.

Exercise 1 Determination of hydraulic conductivity.

Have water entering the model from only the left end (near the injection wells – see figure of the model) and drain out of either the river drain (outlet 1) or the right end (outlet 2).



Measure the discharge (Q) by collecting the draining water over a specified time, then measure the volume of water collected. Q = volume collected/time

Volume collected _______________

Time during which water collected_______________

Q _______________

Measure the gradient (slope) at that discharge rate. This is the difference in the level of water in wells A and B divided by the distance between those two wells. You could also use the difference between water levels at the recharge and discharge points divided by the distance between those two points. Note: the slope will be a negative number for this exercise.

height of water at a (h_a) ____________

height of water at b (h_b) ____________

horizontal distance between the wells (dl=b-a) _____________

gradient = $(h_b-h_a)/dl$ = ______________________

Measure the cross-sectional area of the model aquifer. Only measure the part of the aquifer that is conducting water. The unsaturated portion and the clay are not part of the aquifer so they shouldn't be included. The height varies some so you will have to decide how to estimate a representative height.

width of model aquifer (w) ________________

height of model aquifer (h) ________________

cross-sectional area = A=(w)(h) = _________________

Solve Darcy equation for K. K= -Q/iA

For example,

Q = 100 ml in 10 minutes = 100cm^3/600sec = 0.167 cm^3/sec

i = 1 cm/12 cm = 0.083

A = 2.54cm x 25.4cm = 64.5cm^2

K = -Q/iA = (0.167 cm^3/sec)/(0.083 x 64.5cm^2) = 0.031 cm/sec =3.1 x 10^{-2}cm/sec

This is right in the range of hydraulic conductivity for well-sorted sand on the table below.

Hydraulic Conductivity Ranges of Geologic Materials

Unconsolidated sediments	K cm/sec	Consolidated Rock	K cm/sec
Clay	$10^{-9} – 10^{-6}$	Shale	$10^{-10} – 10^{-7}$
Silt	$10^{-6} – 10^{-4}$	Fractured Igneous Rock	$10^{-6} – 10^{-2}$
Fine Sand	$10^{-5} – 10^{-3}$	Limestone	$10^{-7} – 10^{-3}$
Well Sorted Sand	$10^{-3} – 10^{-1}$	Sandstone	$10^{-8} – 10^{-4}$
Gravel	10^{-2} - 1	Karst Limestone	10^{-4} – 1

Measurements can be done in any unit and then converted using the conversion table shown below. Be sure to keep track of units and be consistent in using units through a particular exercise. Hydrogeologists typically work in units of centimeters per second (cm/s) or feet per day(ft/day).

To convert from:	Multiply by:	To get:
Feet	0.3048	Meters
Meters	3.28	Feet
Inches	2.54	Centimeters
Centimeters	0.3937	Inches
Feet	30.48	Centimeters
Centimeters	0.0328	Feet
Gallons	0.1337	Feet^3
Feet^3	7.48	Gallons

K = 3.1 x 10^{-2}cm/sec x 86400sec/day x 0.0328 ft/cm = 85ft/day

Additional options

Raise one end of the model an inch or so, determine the new values and redo above calculations. Or restrict the flow out of the discharge tube by raising it up, thereby reducing both the Q and the gradient.

How do the values for K compare? *(The values should be very close if not identical.)*

Why are they similar? *(K is a property of the aquifer material and not dependent on gradient.)*

You can determine K for the confined system by using a siphon to pull water out of the artesian pumping well to determine Q, measuring the gradient and cross sectional area of the artesian aquifer and redoing the calculations.

Groundwater velocity

The velocity of groundwater flow in an aquifer system can be determined for the model system. Velocity is a function of the conductivity (K), the gradient (i) and the effective porosity or interconnectedness of the pore spaces (ne). Q/A would be the velocity of the water if the aquifer was all water or moving through a pipe. Because the sand takes up space, only the area of the pore space, ne, is available for the water to move through. The velocity of the water must increase by 1/ne times so that Q amount of water is still moving through the sand. The following equation will give an estimate of groundwater flow velocities.

$V = Q/An_e = -Ki/n_e$

Where:

V = velocity (length/time)

K = hydraulic conductivity (length/time)

i = gradient (unitless)

n_e = effective porosity (use .30 for medium sand, .33 for coarse sand, .27 for fine sand) (unitless)

For example,
From exercise 1
K= 85ft/day
i = 0.083
n_e = 0.30

V= (85ft/day x 0.083)/ 0.30 = 24 ft/day

Effective Porosity (n_e) of common aquifer materials

Unconsolidated sediments	n (%)	Consolidated rocks	n (%)
Clay	45-55	Sandstone	5-30
Silt	35-50	Limestone	1-20
Sand	25-40	Shale	0-10
Gravel	25-40	Granite	0-10
Sand and Gravel mixes	10-35	Basalt	10-50
Glacial till	10-25		

V=Ki/ne =____________________

Exercise 2 Predicting groundwater travel time and velocity
Measure the distance from an injection well to a discharge point (or some other intermediate point) and predict the time it will take for the dye to make it to that point.
Predicted time = distance/velocity. Use the velocity you calculated above.

Measured distance from injection well to discharge point _________________
Estimated time = measured distance/velocity = ________________________
Inject dye and monitor time it takes for dye to reach point. ____________________
How was the prediction?

Why might there be a difference? *(attenuation, diffusion/dispersion, preferential flow, estimate of n_e is off, n_e will change over the life of the model due to biofilm development and settling in transport, gradient different than the water table used to calculate K)*

Inject and measure the time it takes for the dye to reach an actively pumping well. Measure the new gradient. Is it faster or slower than the nonpumping situation? *(This will be faster due to the increased gradient. K and n_e remain the same.)*

What implications does this have for water management? *(Pumping will accelerate the movement of groundwater and any contaminants toward the pumping well. Pumping may divert groundwater that would normally discharge to a river, lake or wetland, thereby reducing the flow in that surface water body.)*

Other options

Find or make a water table map (contours of head) of your area or use the one from the activity Go with the Flow (from the Groundwater Study Guide) and measure the gradient (dh/dl). Have the students draw a flow path (perpendicular to head contours) and calculate how long it would take for contamination from a landfill or septic tank (pick a source) to reach a well or the river. Choose different geologic materials and see what difference it makes in the travel time. For each different geologic material, you'll need to select an appropriate hydraulic conductivity and effective porosity from the tables above. For which geologic material is the travel time longest? For which is it shortest?

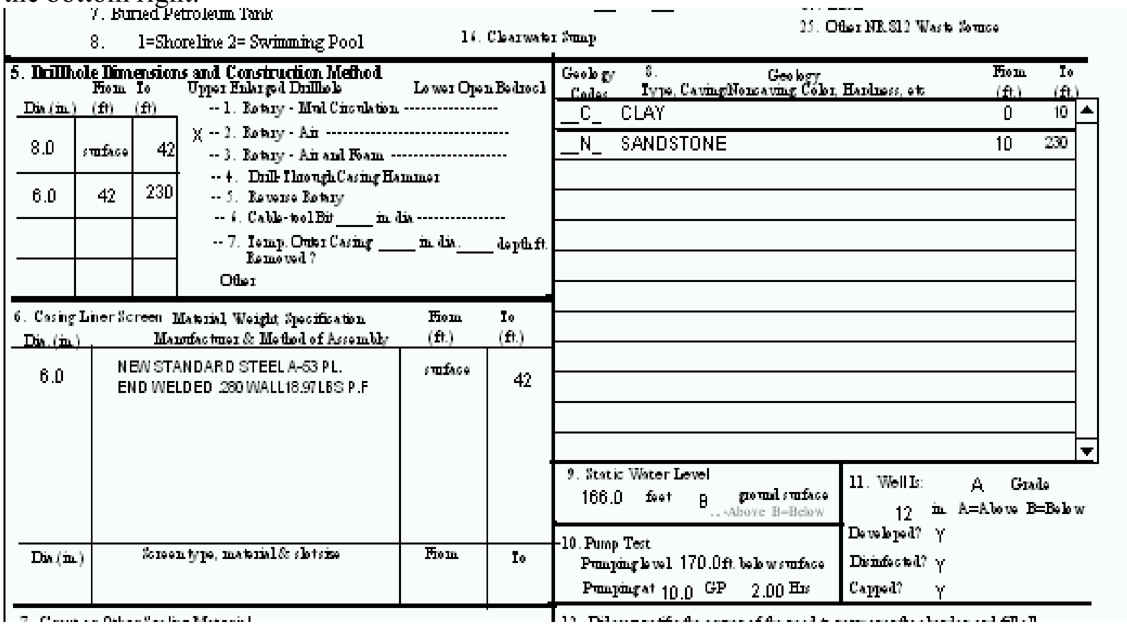
The section below is for advanced students

Aquifer Transmissivity

Much of the time groundwater flows horizontally so we can rewrite the equation for flow as Q=KiA=Kibw where b is the aquifer thickness and w is the aquifer width. We can lump K and be together as transmissivity, T=Kb so the Q=Tiw. Transmissivity is the ability of an entire aquifer to transmit water. The best way to determine T is to pump a well and measure water levels in other monitoring wells - more data than is usually available. Another way to estimate T is by using pump test data from a well construction report (WCR). These WCRs are required for all wells in Wisconsin. On the bottom of the WCR are data on the static water level, pumping water level, and pumping rate. The ratio of pumping rate over the drawdown, the difference between static and pumping water levels, is the specific capacity, Sc, of a well.
Sc=Q/(static level-pumping level).

A well that is in a high transmissivity aquifer will have little drawdown during pumping so it will have a high specific capacity. A well in a low transmissivity aquifer will have a large amount of drawdown during pumping so it will have a low specific capacity. A general empirical rule relating specific capacity to transmissivity is:
T=Sc x 2000 where T is in gpd/ft and Sc is in gpm/ft.
The factor of 2000 contains the conversion from gpm/ft to gpd/ft and would need to be adjusted for other units.

This is the lower portion of a well construction report, which has the pump test information at the bottom right.


7. Buried Petroleum Tank

8. 1=Shoreline 2= Swimming Pool

14. Clearwater Sump

25. Other NR 812 Waste Source

5. Drillhole Dimensions and Construction Method

Dia (in.)	From (ft)	To (ft)
8.0	surface	42
6.0	42	230

Upper Enlarged Drillhole / Lower Open Bedrock

-- 1. Rotary - Mud Circulation

X -- 2. Rotary - Air

-- 3. Rotary - Air and Foam

-- 4. Drill-Through Casing Hammer

-- 5. Reverse Rotary

-- 6. Cable-tool Bit ____ in. dia

-- 7. Temp. Outer Casing ____ in. dia. ____ depth ft. Removed?

Other

Geology Codes	8. Geology Type, Caving/Noncaving, Color, Hardness, etc.	From (ft.)	To (ft.)
__C_	CLAY	0	10
__N_	SANDSTONE	10	230

6. Casing Liner Screen

Dia. (in.)	Material, Weight, Specification, Manufacturer & Method of Assembly	From (ft.)	To (ft.)
6.0	NEW STANDARD STEEL A-53 PL. END WELDED .280 WALL 18.97 LBS P.F	surface	42

Dia (in.)	Screen type, material & slot size	From	To

9. Static Water Level 166.0 feet B ground surface A=Above B=Below

11. Well Is: A Grade 12 in. A=Above B=Below

Developed? Y

Disinfected? Y

Capped? Y

10. Pump Test

Pumping level 170.0 ft. below surface

Pumping at 10.0 GP 2.00 Hrs

For example, using information from this well construction report (WCR)

Pump rate = 10 gpm

Drawdown =Pumping Level-Static Level= 170-166 = 4 ft

Sc = 10 gpm/4 ft= 2.5gpm/ft

T = Sc x 2000 = 2.5 x 2000 = 5000 gpd/ft (use the conversion in the table to go from gallons to ft^3)

5000gpd/ft/7.48gals/ft^3 = 668.5 ft^2/day

Exercise 3 Estimating Transmissivity

What is the transmissivity of the model?

Example calculations:Transmissivity is K x b, where K, from Exercise 1, is in feet per day and b is in feet (approximately 0.8 feet).

T =(85ft/d)(0.8ft) = 68 ft^2/d x 7.48 gals/ft^3 = 509gpd/ft

In other words if the model were to run all day and the discharge was the entire right side of the model, 509 gallons would discharge from the model in one day (24 hours).

Find several well logs from your area and estimate the T from the pump test data. Or look at the WCR info on your city's public well at http://www.dnr.state.wi.us/org/water/dwg/DWS.htm. Choose public systems and type in city name, select water supply system by name then scroll down and choose entry points. Click on a well number and then chose well construction data, the specific capacity for the well is near the bottom right of the form before the tables on casing and geology.

Why do the Ts differ for wells in the area? (*The reason transmissivity differs is related primarily to the hydraulic conductivity of the aquifer in the immediate area around the well. In sand and gravel wells one well might hit a larger more continuous seam of high porosity material and thus have a higher K and resultant higher T. In bedrock wells a major influence on K and thus T would be fracturing or bedding planes in the rock that allow for increased flow. Transmissivity will also be affected by the depth the well penetrates into the aquifer and to some degree the well efficiency.)*

What is the hydraulic conductivity, K, of the municipal wells in your area?
Using the estimated T for your municipal well, convert it to ft^2/d and divide by the thickness of the aquifer to find K. If the Sc of the well were 12.5 gpm/ft then the estimated T would be 12.5 x 2000 = 25000 gpd/ft. Then convert this to ft^2/d by dividing by 7.48 $gals/ft^3$ = 3342.25 ft^2/d. If the aquifer, from the water table to the bottom of the well is 290 feet then the average K of the aquifer is 11.5 ft/d.

TGUESS Program
A computer can also be used to estimate transmissivity. Tguess is a program written by Ken Bradbury and E. R. Rothschild which uses basic information from a well construction report or other pump test data to estimate the transmissivity of the aquifer being tested. The program is available from the Wisconsin Geological and Natural History Survey and from the International Ground Water Modeling Center in Golden, CO. The program code is available in the March-April 1985 issue of Groundwater.

References
The following paper is downloadable from the web and has more information on the concepts of groundwater flow.
Basic Ground-Water Hydrology, RC Heath, USGS Water Supply Paper 2220
http://water.usgs.gov/pubs/wsp/wsp2220/

Groundwater, Freeze and Cherry, 1979, Prentice Hall

Applied Hydrogeology, Fetter, 1988, Merrill

Groundwater and Wells, Driscoll, 1986, Johnson Filtration Systems