SEX-AGE-KILL FORMULA

The SAK is a herd reconstruction technique for determining the population immediately prior to the harvest season. Originally developed by Eberhardt (1960) in Michigan, the **count** in this case is the registered buck kill and the **probability of detection** is the proportion of the prehunt population of deer represented by the registered buck kill. Considerable effort is expended in trying to estimate probability of detection and how it varies in time and space. The details of methods used have, and will continue to, evolve over time, but the basic model remains constant. Postseason estimates are determined by subtracting known harvests and an estimate of unretrieved/uncounted (i.e., accidental/non-registered) loss.

Simple Conceptual Framework

The SAK procedure for estimating deer populations can be explained on several different levels. Perhaps the simplest explanation is the "Boys and Girls" example (McCaffery, unpublished):

> Like boys and girls, male and female deer are born in approximately equal numbers. By aging harvested deer, we find that adult bucks die about twice as fast as does. Thus, the prehunt adult sex ratio is about 2 does per buck. Typically, net fawn recruitment in fall averages about 1 fawn per doe. Therefore, for every adult buck in the fall herd we typically have 2 adult does, 1 buck fawn, and 1 doe fawn. At the start of the fall hunt, 1 deer out of 5 will carry antlers.
>
> If all of the adult bucks were harvested and registered, one could multiply the buck harvest by 5 (proportion of the prehunt population of deer represented by the registered buck kill = 0.2) to estimate the prehunt population. Obviously not all bucks are harvested each fall and some eventually die from a variety of nonharvest causes. If only half of the bucks are harvested, you would multiply the buck kill by 10 (proportion of the prehunt population of deer represented by the registered buck kill = 0.1). Sex, age, and production information obtained each year enables us to calculate this multiplier. SAK is merely the method used to determine this multiplier for each unit each year.

Mathematical Calculations

The following explanation describes how sex, age, and production information are combined to estimate the multiplier, and in turn, population size. The mathematics are described in detail to illustrate the logic underlying the model. The assumptions associated with the model are discussed in the following section.

The SAK procedure has 3 major steps: 1) the number of **adult bucks** in the fall population is estimated, 2) the number of **does** is estimated, and 3) the number of **fawns** is estimated. Each of the 3 segments of the population is estimated using a different procedure.

1) Bucks

Mandatory deer registration provides highly accurate counts of the size of the buck harvest for each deer management unit. Since harvest is known, if we can estimate the harvest rate of bucks then we can estimate the size of the pre-hunt buck population. By definition, **harvest rate** is:

$$\textbf{harvest rate of bucks} = \frac{\text{number of bucks harvested}}{}$$

number of bucks in the population

So if harvest rate can be estimated we can solve for **prehunt buck population**:

$$\frac{\text{number of bucks harvested}}{\text{harvest rate of bucks}} = \textbf{number of bucks in the population}$$

For example, in Deer Management Unit 58, 3,014 bucks were taken in the 1997 hunting season. If the buck harvest rate was 0.64 then the buck population before the start of the hunting season can be estimated as 3,014 ÷ 0.64 = 4,709.

It is possible to estimate buck harvest rate directly using tagged or radio-collared animals. However, in the SAK method we estimate buck harvest rate indirectly by multiplying (1) an estimate of buck mortality rate by (2) an estimate of the proportion of buck deaths due to legal harvest.

Mortality Rate = number bucks dying each year from all causes ÷ number bucks in population

Proportion of buck deaths due to legal harvest = number dying from harvest ÷ number bucks dying from all causes

When these 2 proportions are multiplied together, the number of bucks dying from all causes cancels out:

$$\frac{\cancel{\text{\# dying all causes}}}{\text{\# in population}} \times \frac{\text{\# dying (harvest)}}{\cancel{\text{\# dying all causes}}}$$

leaving an estimate of buck **harvest rate**:

dying (harvest) ÷ # in population = **Harvest Rate**

This indirect method of estimating harvest rate is used because one of the parameters, mortality rate of bucks, can be estimated from the age structure of the population. Harvest provides considerable information on population age structure. The percentage of yearlings in the population of adult bucks provides an estimate of mortality rate. To understand why the percentage of yearlings is an estimate of mortality rate, consider the following example:

> Assume a population of 1,000 adult bucks in which 50% die each year. If this population were to persist for long, 500 yearlings would have to be added each year to replace the 500 adults that die each year. The 1,000 adults would consist of 500 yearlings and 500 older bucks. Yearlings would comprise 50% of the adult population, and the percentage of yearlings would equal the mortality rate.

While mortality rate can be estimated from ages of harvested deer, converting this estimate to an estimate of harvest rate requires an estimate of the proportion of buck deaths that are due to legal harvest. The proportion of deaths due to legal harvest has been referred to as the Buck Recovery Rate (BRR). In heavily hunted populations, relatively few bucks live past the hunting season to be exposed to other mortality sources and the vast majority (~ 90%) of buck deaths are due to legal harvest. In more lightly hunted populations, more bucks survive the hunting season and greater numbers are exposed to other causes of mortality. In these populations the proportion of deaths due to legal harvest may be as low as 60%.

If we multiply our estimates of buck mortality rate by the proportion of deaths due to harvest we can estimate the buck harvest rate. We can then divide the actual buck harvest by this estimate of buck harvest rate to estimate the size of the buck population prior to the hunting season.

$$\frac{\text{Buck Harvest}}{\text{Buck Harvest Rate}} = \textbf{Prehunt Buck Population}$$

Unit 58 Example: Aging data shows about 75% of the bucks in the population before the hunting season are yearlings. We estimate that hunting is the cause of death for about 85% of adult bucks in that unit. Multiplying these 2 estimates together (0.75 x 0.85) gives an estimate of the buck harvest rate (0.64). As shown above, dividing the buck harvest by the buck harvest rate (3,014 ÷ 0.64) yields the prehunt estimate of 4,709 bucks.

2) Does

If we have an estimate of the size of the buck population we can multiply this estimate by the adult sex ratio (number of adult does per adult buck in the population) to estimate the size of the doe population. However, it has been consistently shown that sex ratios estimated from direct observations of deer are biased because of sex related differences in habitat use and activity patterns (Downing et al. 1977, McCullough 1982). Therefore, we use an indirect method — the ratio of the proportion of adult bucks that are yearlings (YB%) and the proportion of adult does that are yearlings (YD%) — to estimate the adult sex ratio. While it is not intuitively obvious why the ratio of these two proportions should yield an estimate of the adult sex ratio, if we work through the math we should see why this works.

YB% = number of yearling bucks ÷ total number of adult bucks (yearlings and older).

YD% = number of yearling does ÷ number of adult does (yearlings and older).

The ratio of these two proportions is:

$$\frac{\left(\dfrac{\text{number of yearling bucks}}{\text{number of adult bucks}}\right)}{\left(\dfrac{\text{number of yearling does}}{\text{number of adult does}}\right)}$$

Now, if we assume that (1) deer give birth to equal numbers of doe and buck fawns, and (2) the causes of fawn mortality (predation, starvation, deer-vehicle accidents, etc.) do not discriminate among sexes, then there would be equal numbers of yearling bucks and does in the population prior to the fall hunting season. In this case, we can substitute the number of yearling bucks for the number of yearling does in the above ratio of proportions:

$$\frac{\dfrac{\text{number of yearling bucks}}{\text{number of adult bucks}}}{\dfrac{\text{number of yearling bucks}}{\text{number of adult does}}}$$

The ratio can be rewritten as:

$$\frac{\text{number of yearling bucks}}{\text{number of adult bucks}} \times \frac{\text{number of adult does}}{\text{number of yearling bucks}}$$

because dividing the top proportion by the bottom proportion is the same as multiplying the top proportion by the inverse of the bottom proportion.

Now because the number of yearling bucks is in the numerator of one term and in the denominator of the other, it cancels out:

$$\frac{\cancel{\text{number of yearling bucks}}}{\text{number of adult bucks}} \times \frac{\text{number of adult does}}{\cancel{\text{number of yearling bucks}}}$$

and we are left with:

$$\frac{\textbf{number of adult does}}{\textbf{number of adult bucks}}$$

which is the adult sex ratio we are trying to estimate.

Actually, examination of nearly 2,000 fetuses from vehicle-killed deer in Wisconsin during the 1982-87 found that slightly more male fawns are born than female fawns, about 110 fetuses were male for every 100 that were female. Therefore, we adjust the above formula to account for an unbalanced sex ratio at birth, assuming this imbalance persists through the 18 months of life.

Unit 58 Example: About 75% of adult bucks are yearlings. Aging data finds that 40% of adult does are yearlings. The ratio of these 2 percentages, adjusted for the sex ratio at birth [(75 ÷ 40) x (100 ÷ 110)], is our estimate of the adult sex ratio (1.70 does per buck). We can multiply our estimate of the adult sex ratio by our estimate of the buck population to estimate the number of does in the fall population. In Unit 58 with 4,709 bucks x 1.70 does per buck = 8,005 does in the fall 1997 population.

3) Fawns
We can estimate the number of fawns alive before the hunt by multiplying the estimate of the number of does by the ratio of fawns per doe in the population. For the forested regions of Wisconsin we have relied on direct observations of deer from the Summer Deer Observation Survey to estimate the fawn:doe ratio. DNR personnel and other cooperators annually observe about 4,000 does while driving in these areas. On average, approximately 1 fawn is seen for every doe. This ratio varies from region to region and from year to year.

In the farmland regions we have assumed fixed, long-term fawn:doe ratios (based on yearling doe percents) in the SAK estimates because annual variation in observed ratios is not consistent with other indices of recruitment. The ratios used are dependent on recruitment levels reflected in the percentage of does that are yearlings. This is discussed in more detail below.

Unit 58 Example: The estimated fawn:doe ratio for 1997 was 0.71, lower than average due to the severe winter of 1996-97. Multiplying this by the estimate of the doe population (8,005) results in a fall 1997 population estimate of 5,684 fawns.

SAK Prehunt Estimate

With estimates for the 3 parts of the deer population, we can estimate the size of the prehunt deer population by adding together the 3 separate estimates.

Unit 58 Example: 4,709 bucks + 8,005 does + 5,684 fawns = 18,398 deer in the Unit 58 population before the start of the 1997 hunting season. Recall that our **count** of deer was the registered buck kill in Unit 58, which equalled 3,014. The proportion of the population represented by the buck kill, was 3,014 ÷ 18,390 = 0.164. So the prehunt population estimate = 3,014 ÷ 0.164. In other words, the preseason population can be estimated by multiplying the buck harvest by 6.1 (3,014 × 6.1 = 18,398).

SAK Posthunt Estimate

Our population goals are expressed as a density of deer remaining after the hunting season (deer/mi^2 of deer range). To compare our population estimates to these goals we estimate a posthunt population size by subtracting the known total harvest, and an estimate of unretrieved/uncounted loss from the prehunt population estimate. We have been assuming that poaching and wounding losses are 15% of the registered harvest. To estimate the posthunt density, we divide the posthunt population estimate by an estimate of the area of deer range in the unit.

Unit 58 Example: 18,398 deer prehunt - 6,043 registered harvest - 906 unretrieved/uncounted deer = 11,449 deer posthunt. 11,449 deer posthunt ÷ 464 mi^2 deer range = 25 deer/mi^2.

Worksheet Procedures

The following worksheet illustrates all of the calculations used in SAK estimates. The sheet facilitates manual calculation of population estimates and can be used for instructional purposes with clients.

SAK population estimates are an approximation of the actual population. Some discretion or interpretation may be necessary when applying the SAK to the management situation. While all input values are subject to error, the most notable input affecting the index on an annual basis is the buck recovery rate. The SAK relies heavily on the assumption that the annual buck harvest rate is relatively consistent from year to year for individual units. Adjustments may be necessary in cases of extreme hunting conditions (see below).

Compensating for variations in buck harvest rate should be done only when there are data to support it. How much to compensate is a judgment call based on experience. The Bureau of Wildlife Management approved SAK is the official DNR population estimate. Annual SAK estimates statewide are provided by midwinter. A narrative explanation will accompany the estimates.

SAK WORKSHEET

Year ________ Deer Management Unit _________ Deer Range = _________

1a) Total Buck Harvest _______ 1b) Total Harvest (bucks + antlerless) _______

BUCKS

2) % of bucks that are yearlings (Mortality Rate, AYB%) = ________

3) % deaths due to hunting (BRR)= ______

4) Harvest Rate = Mortality Rate (2) x BRR (3) = ______ x ______ = _______

5) Fall Buck population = Registered buck kill (1a) ÷ Harvest Rate (4) =
_______ ÷ _______ = _________ bucks

DOES

6) % of adult bucks that are yearlings (AYB%) = _(2)____

7) % of adult does that are yearlings (AYD%) = ________

8) Adult Sex Ratio =

$$\frac{AYB\%(6)}{AYD\%(7)} \times \frac{100}{110} = \_\_\_\_\_\_ / \_\_\_\_\_ \times 100/110 = \_\_\_\_\_ \text{ Does/buck}$$

9) Fall Doe Population = _______ bucks (5) x _______ does/buck (8) = _________ does

FAWNS

10) Fawn:Doe Ratio = ________

11) Fall fawn population = _____ does (9) x ______ fawns/doe (10) =______fawns

PREHUNT POPULATION

12) Prehunt Popul. = ______ bucks (5) + ______ does (9) + ______ fawns (11) = _______ deer

13) Unretrieved & Uncounted Loss = Registered Total Harvest (1b) x 0.15 =
_______ x 0.15 = _______

POSTHUNT POPULATION

14) **Posthunt Popul.** = Prehunt Pop. (12) - Harvest (1b) - Unretrieved & Uncounted Loss (13) =
________ - ________ - _________ = ________ deer

15) **Posthunt Density** = Posthunt Population (14) ÷ Deer Range = ________ / _________ =
__________ deer/mi^2 of deer range