Reservoir Modeling Overview

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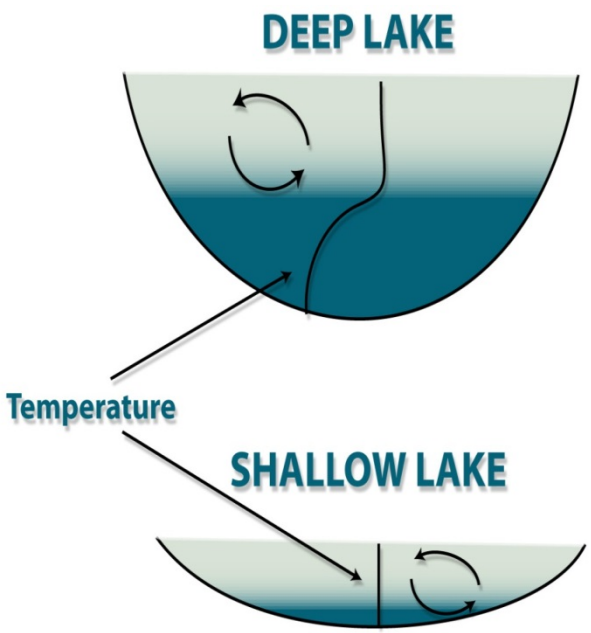
Physical Characteristics

- Mixing / stratification
- Lake depth
- Residence time / flushing rate
- Drainage basin/ lake area ratio
- Landscape position
- Influence of watershed runoff

Lake Depth Matters

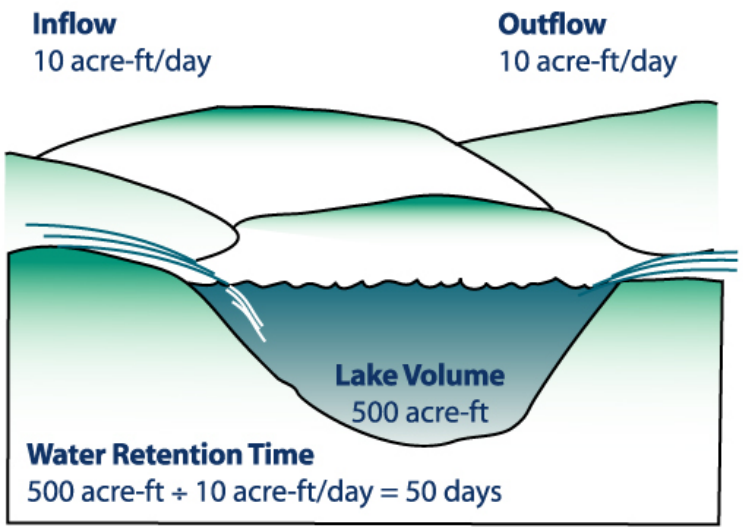
- Deep Lakes
  - Stratify
- Shallow Lakes
  - Continuous Nutrient Recycling



Residence Time/Flushing Rate

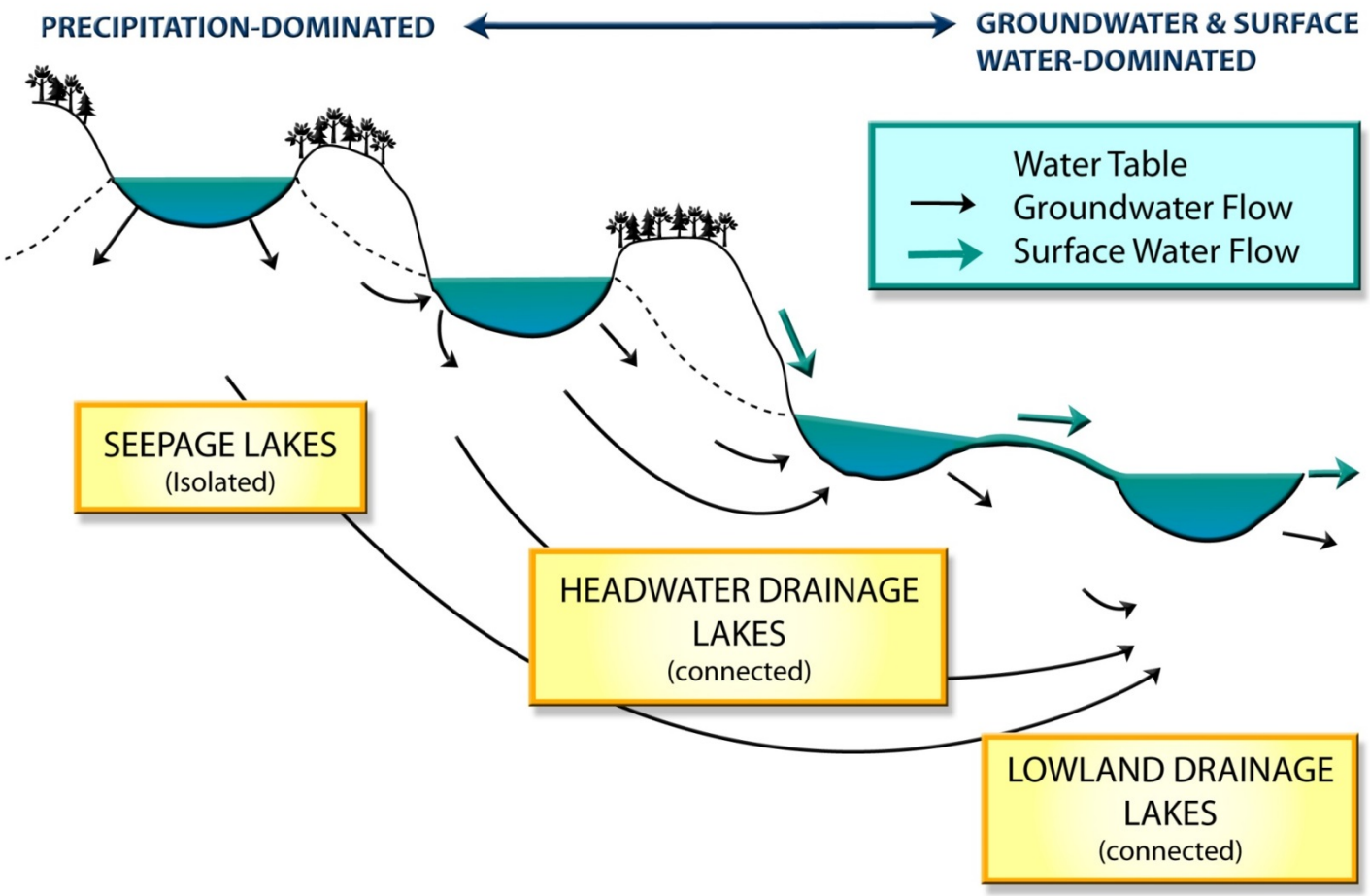
- How long would it take to fill a lake?
- Residence time matters

Reservoir	Mean Summer Residence Time (days) [1]
Big Eau Pleine	464
DuBay	14
Petenwell	69
Castle Rock	19
Lake Wisconsin	8



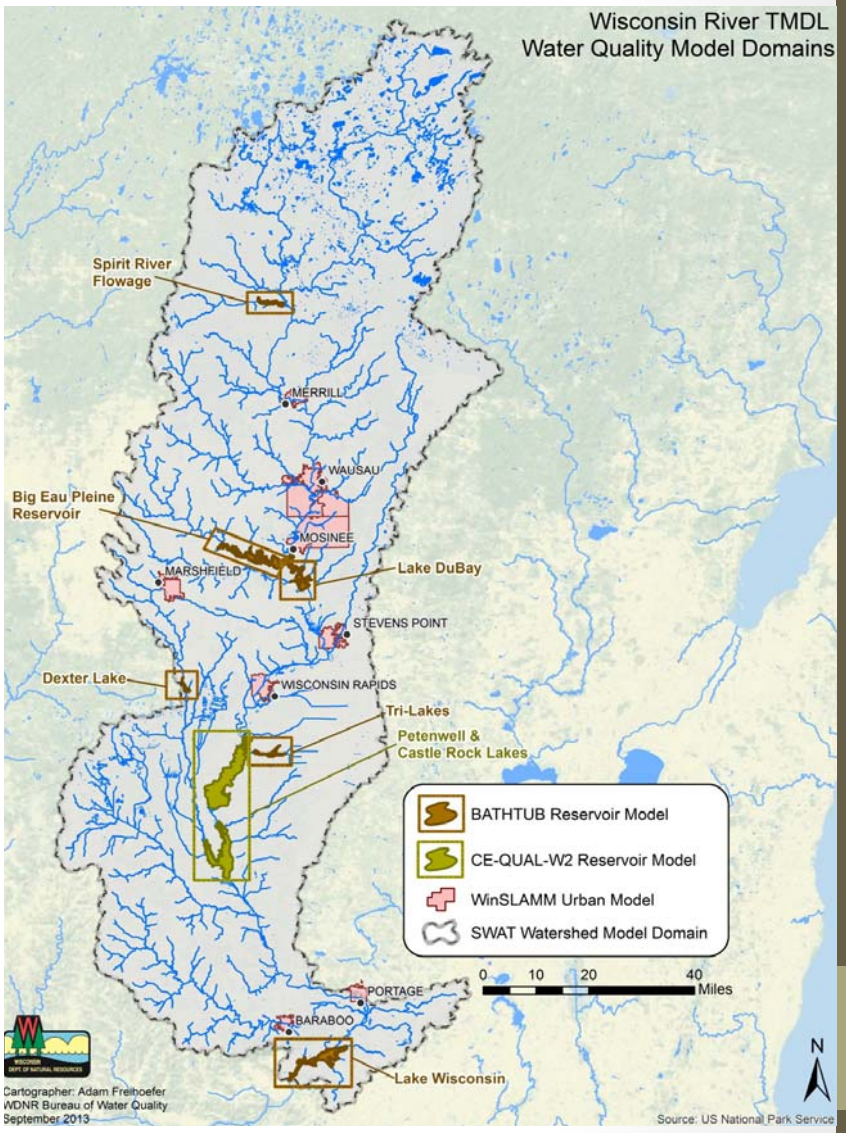
[1] Conservative (high) preliminary estimate

Landscape Position



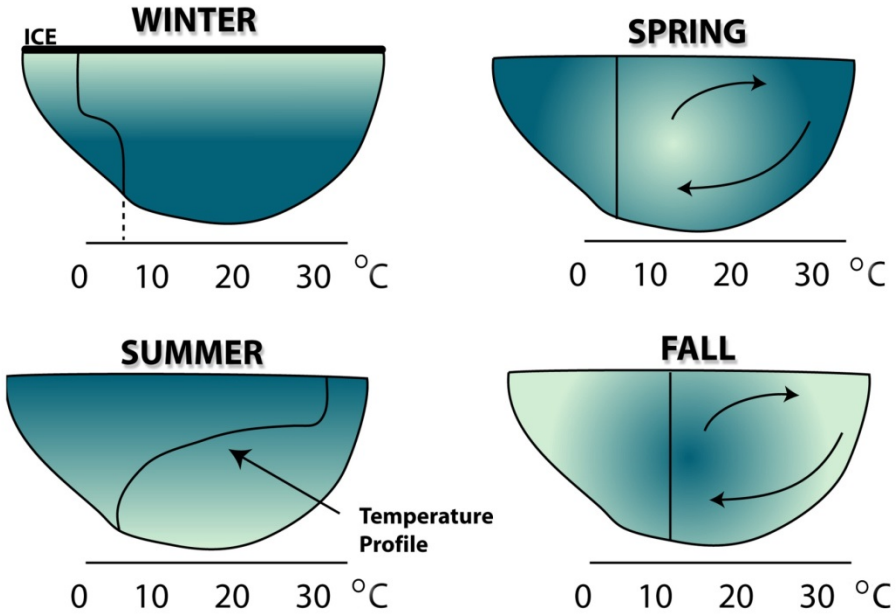
We're at the bottom at the hill boys and girls…

- Reservoirs are Particularly Vulnerable
  - Larger watersheds
  - Larger land-water interface
  - Shallower depths
  - Often built in eutrophic landscapes



Chemical Characteristics

- Nutrients
  - Phosphorus
  - Nitrogen
- Dissolved Oxygen
- Hardness/ Alkalinity
- pH
- Many more…



NUTRIENT FUNCTIONS

ELEMENT	AVAILABILITY	DEMAND	AVAILABILITY DEMAND	FUNCTION
Na	32	0.5	64	Cell membrane
Mg	22	1.4	16	Chlorophyll, energy transfer
Si	268	0.7	383	Cell wall (diatoms)
P	1	1	1	DNA, RNA, ATP, enzymes
K	20	6	3	Enzyme activator
Ca	40	8	5	Cell membrane
Mn	0.9	0.3	3	Photosynthesis, enzymes
Fe	54	0.06	900	Enzymes
Co	0.02	0.0002	100	Vitamin B12
Cu	0.05	0.006	8	Enzymes
Zn	0.07	0.04	2	Enzyme activator
Mo	0.001	0.0004	3	Enzymes

Chemical Characteristics

- Phosphorus is nearly always the primary limiting nutrient in freshwater systems
- Key question: how does a lake respond to phosphorus loading?

Start Simple: Mass Balance

$$accumulation = inputs - outputs - sedimentation$$

Just a couple of (good) ways to look at it-

$Vollenweider$ (1969)

$$V\frac{dP}{dt} = \mathrm{W} - \mathrm{QP} - \sigma VP$$

$Chapra$ (1975)

$$V\frac{dP}{dt} = \mathrm{W} - \mathrm{QP} - v_s A_s P$$

Where:
P = lake phosphorus concentration $[M/L^3]$
W = mass rate of phosphorus inflow $[M/T]$
Q = volume rate of water inflow $[L^3/T]$
V = lake volume $[L^3]$

σ = the sedimentary loss rate coefficient $[T^{-1}]$

v_s = apparent settling velocity $[L/T]$
A_s = lake surface area $[L^2]$

Mass Balance at Steady State:

Vollenweider (1969)

$$P = \frac{W}{Q + \sigma V} = \frac{L}{z({}^{1}/_{\tau_w} + \sigma)}$$

Chapra (1975)

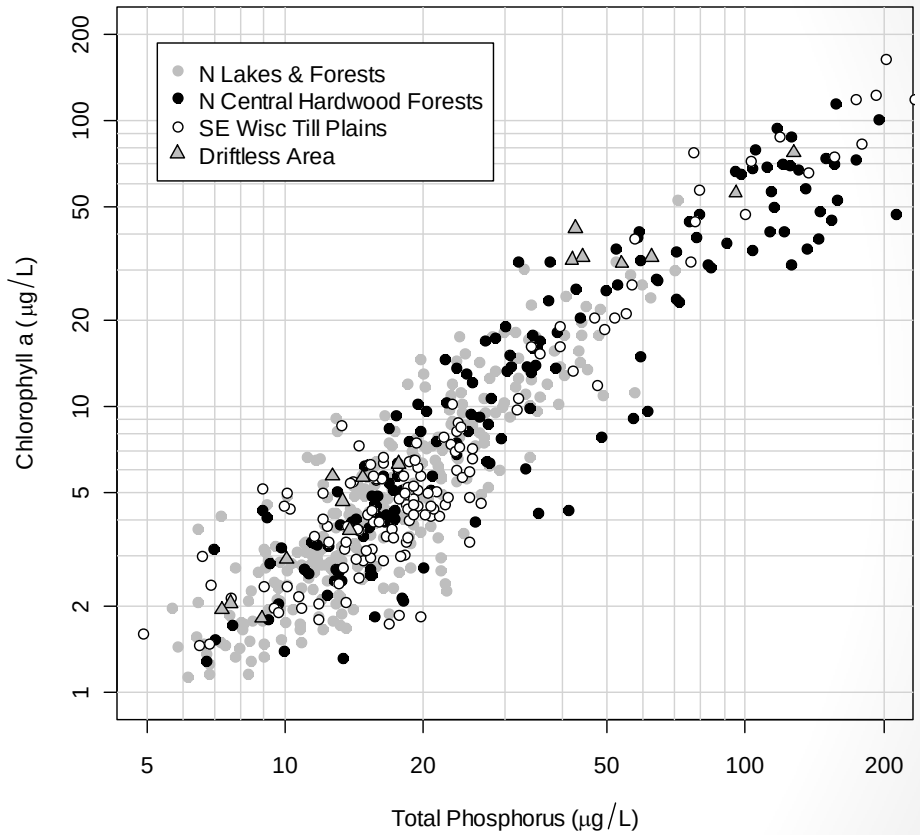
$$P = \frac{W}{Q + v_s A_s} = \frac{L}{({}^{z}/_{\tau_w} + v_s)}$$

Where:
L = areal phosphorus loading $[M/L^2T]$
z = lake mean depth $[L]$
τ_w = hydraulic residence time $[L^3/T]$

σ and v_s are inherently <u>empirical</u> as they are represent very complex interactions

So why choose empirical lake modeling?

- Works well in most circumstances
- Error and uncertainty can be easily quantified
- Input parameters are easily quantified
- Lower monitoring and modeling costs



Well, what about when it doesn't work?

- What we said:

$$V\frac{dP}{dt} = \mathrm{W} - \mathrm{QP} - \sigma VP \quad \text{and/or} \quad V\frac{dP}{dt} = \mathrm{W} - \mathrm{QP} - v_s A_s P$$

- Remember: σ and v_s represent very complex interactions
- What really is happening:

$$V\frac{dP}{dt} = W_I - W_o - \emptyset(W_I, W_o, P, Ca, Fe, Al, pH, \tau_w, z, V, \ldots)$$

Where:
W_I = mass influx of phosphorus
W_o = mass efflux of phosphorus
$\emptyset$ = net flux of phosphorus to the sediments
… = a whole bunch of other stuff

Not So Simple Steps

- Look at the <u>mechanisms</u> behind the interactions
- Well this is still a math equation:

$$V\frac{dP}{dt} = W_I - W_o - \emptyset(W_I, W_o, P, Ca, Fe, Al, pH, \tau_w, z, V, \dots)$$

- And mass balance principals still apply
- Just the equations just get a little bit more “interesting”…

Not So Simple Steps: Mass Balance

- Hydrodynamics

The laterally-averaged x-momentum equation is more easily simplified by writing it in conservative form (this can be verified by using the continuity equation with the x-momentum equation),

$$\frac{\partial(\overline{\overline{u}}+u'')}{\partial t}+\frac{\partial(\overline{\overline{u}}+u'')(\overline{\overline{u}}+u'')}{\partial x}+\frac{\partial(\overline{\overline{v}}+v'')(\overline{\overline{u}}+u'')}{\partial y}+\frac{\partial(\overline{\overline{w}}+w'')(\overline{\overline{u}}+u'')}{\partial z}$$
$$=\overline{\overline{g\sin\alpha}}-\frac{1}{\rho}\frac{\partial(\overline{\overline{p}}+p'')}{\partial x}+\frac{1}{\rho}\left(\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}\right) \qquad \textbf{(A-27)}$$

Not So Simple Steps: Mass Balance

- Hydrodynamics
- Constituent transport

Note how the following terms are simplified:

$$\frac{1}{B}\int_{y1}^{y2}\frac{\partial(\overline{\overline{c}}+c'')}{\partial t}dy = \frac{1}{B}\int_{y1}^{y2}\frac{\partial(\overline{\overline{c}})}{\partial t}dy + \frac{1}{B}\int_{y1}^{y2}\frac{\partial(c'')}{\partial t}dy$$

$$= \frac{1}{B}\frac{\partial}{\partial t}\int_{y1}^{y2}\overline{\overline{c}}dy + \frac{1}{B}\frac{\partial}{\partial t}\int_{y1}^{y2}c''dy \qquad \textbf{(A-106)}$$

$$= \frac{1}{B}\frac{\partial B\overline{\overline{c}}}{\partial t}$$

Not So Simple Steps: Mass Balance

- Hydrodynamics
- Constituent transport
- Constituent reactions

the rate equation for phosphorus is:

$$
\begin{aligned}
S_p = & \underbrace{\sum (K_{ar} - K_{ag})\,\delta_{Pa}\,\Phi_a}_{\text{algal net growth}} + \underbrace{\sum (K_{er} - K_{eg})\,\delta_{Pe}\,\Phi_e}_{\text{epiphyton net growth}} + \underbrace{K_{LDOM}\delta_{POM}\gamma_{OM}\Phi_{LDOM}}_{\text{labile DOM decay}} \\
& + \underbrace{K_{RDOM}\delta_{POM}\gamma_{OM}\Phi_{RDOM}}_{\text{refractory DOM decay}} + \underbrace{K_{LPOM}\delta_{POM}\gamma_{OM}\Phi_{LPOM}}_{\text{labile POM decay}} + \underbrace{K_{RPOM}\delta_{POM}\gamma_{OM}\Phi_{RPOM}}_{\text{refractory POM decay}} \\
& + \underbrace{\sum K_{CBOD} R_{CBOD}\delta_{P-CBOD}\Theta^{T-20}\,\Phi_{CBOD}}_{\text{CBOD decay}} + \underbrace{K_s\,\delta_{POM}\gamma_{OM}\Phi_s}_{\text{1st-order sediment release}} + \underbrace{SOD\gamma_{OM}\frac{A_{sed}}{V}}_{\text{0-order sediment release}} \qquad \textbf{(B-33)} \\
& - \underbrace{\frac{\left(\sum \varpi_{ISS}\Phi_{ISS} + \omega_{Fe}\Phi_{Fe}\right)P_P}{\Delta z}\Phi_P}_{\text{inorganic solids adsorption}} + \underbrace{\sum (K_{mr} - (1 - f_{psed})K_{mg})\,\delta_{Pm}\Phi_{macro}}_{\text{macrophyte net growth}} + \underbrace{\sum K_{zr}\,\delta_{Pz}\Phi_{zoo}}_{\text{zooplankton respiration}}
\end{aligned}
$$

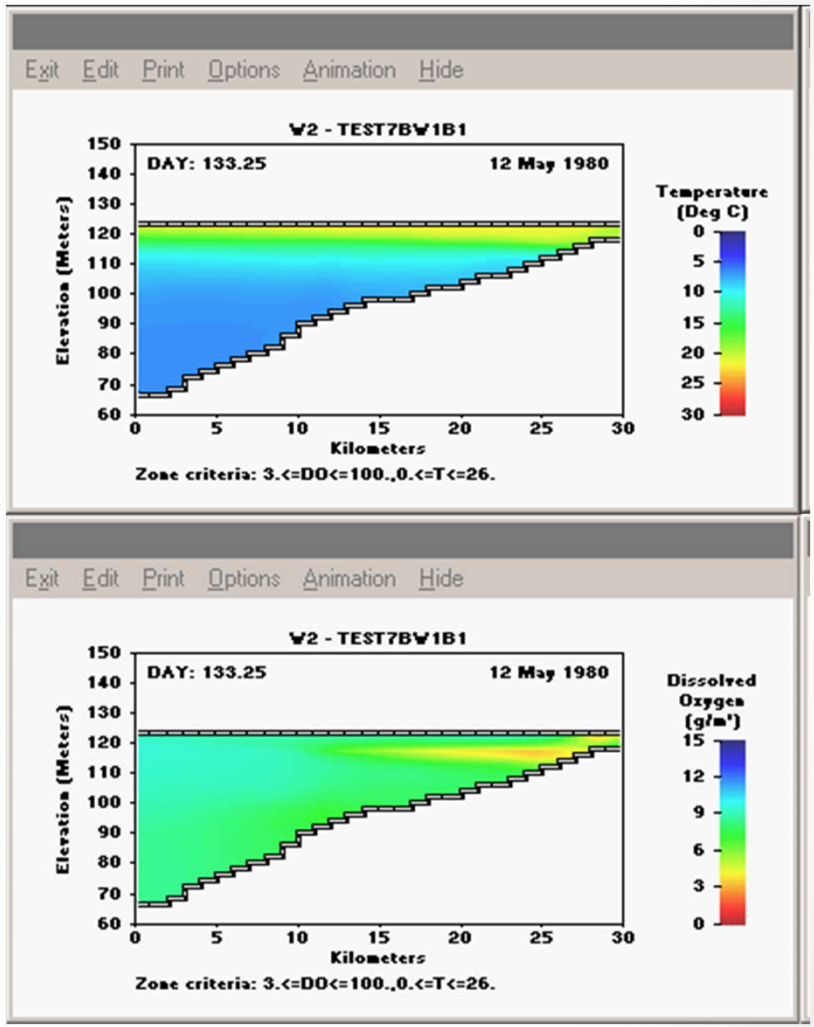
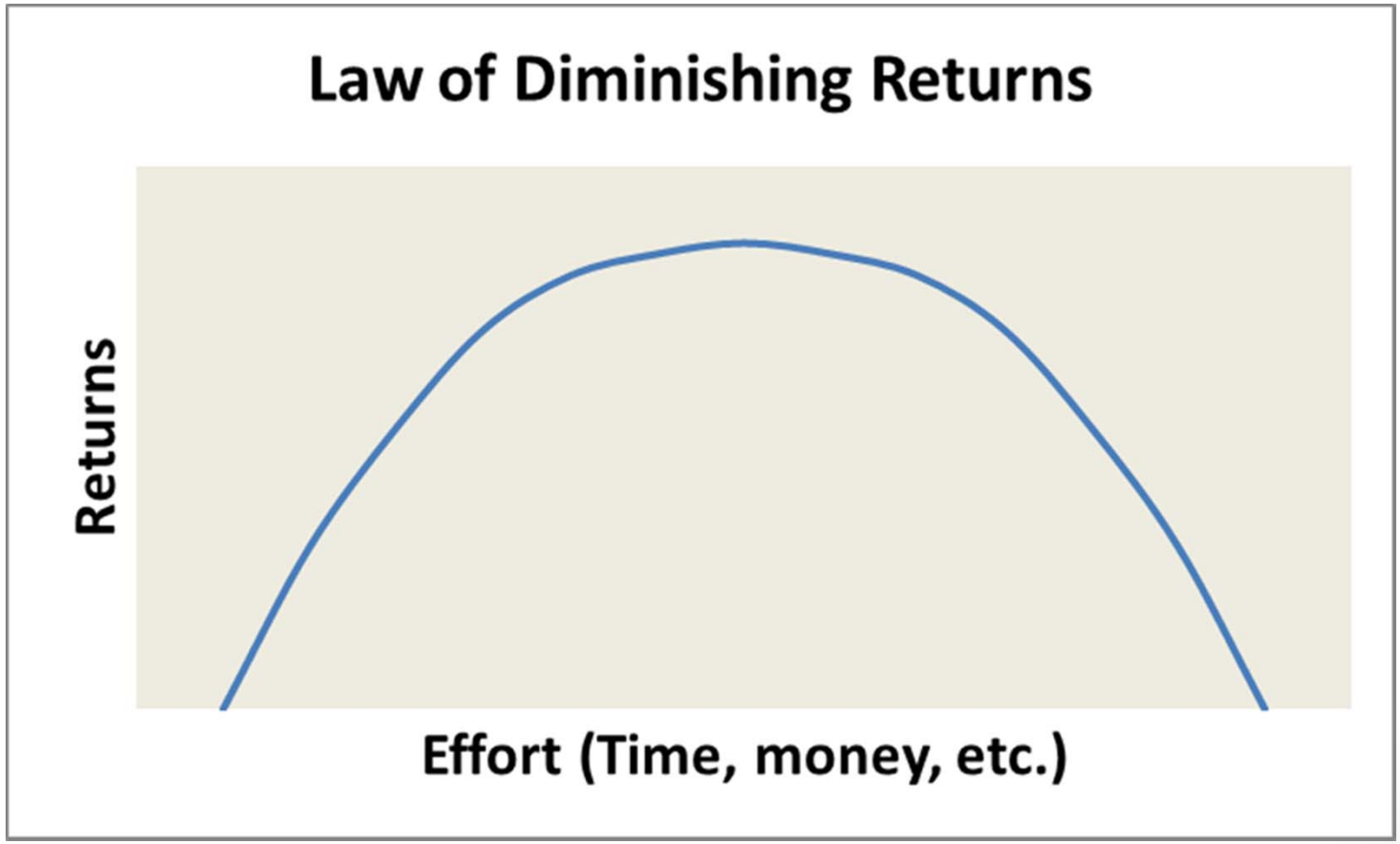
Where:

Δz = model cell thickness, m
A_{sed} = sediment surface area, m^2
V = cell volume, m^3
P_P = adsorption coefficient, $m^3\ g^{-1}$
f_{psed} = fraction of macrophyte phosphorus uptake from sediments
δ_{Pe} = epiphyton stoichiometric coefficient for phosphorus
δ_{Pa} = algal stoichiometric coefficient for phosphorus
δ_{Pm} = macrophyte stoichiometric coefficient for phosphorus
δ_{Pz} = zooplankton stoichiometric coefficient for phosphorus
δ_{POM} = organic matter stoichiometric coefficient for phosphorus
$\delta_{P\text{-}CBOD}$ = phosphorus/CBOD stoichiometric ratio
γ_{OM} = temperature rate multiplier for organic matter decay
Θ = temperature rate multiplier for CBOD decay
R_{BOD} = conversion ratio for 5-day CBOD to CBOD ultimate
ω_{ISS} = inorganic suspended solids settling velocity, $m\ sec^{-1}$
ω_{Fe} = particulate organic matter settling velocity, $m\ sec^{-1}$
K_{ag} = algal growth rate, sec^{-1}
K_{ar} = algal dark respiration rate, sec^{-1}
K_{eg} = epiphyton growth rate, sec^{-1}
K_{er} = epiphyton dark respiration rate, sec^{-1}
K_{mg} = macrophyte growth rate, sec^{-1}
K_{mr} = macrophyte respiration rate, sec^{-1}
K_{zr} = macrophyte respiration rate, sec^{-1}

K_{LDOM} = labile DOM decay rate, sec^{-1}
K_{RDOM} = refractory DOM decay rate, sec^{-1}
K_{LPOM} = labile POM decay rate, sec^{-1}
K_{RPOM} = refractory POM decay rate, sec^{-1}
K_{CBOD} = CBOD decay rate, sec^{-1}
K_{sed} = sediment decay rate, sec^{-1}
SOD = anaerobic sediment release rate, $g\ m^{-2}s^{-1}$
Φ_P = phosphorus concentration, $g\ m^{-3}$
Φ_{Fe} = total iron concentration, $g\ m^{-3}$
Φ_{ISS} = inorganic suspended solids concentration, $g\ m^{-3}$
Φ_a = algal concentration, $g\ m^{-3}$
Φ_e = epiphyton concentration, $g\ m^{-3}$
Φ_{LDOM} = labile DOM concentration, $g\ m^{-3}$
Φ_{LPOM} = labile POM concentration, $g\ m^{-3}$
Φ_{RDOM} = refractory DOM concentration, $g\ m^{-3}$
Φ_{RPOM} = refractory POM concentration, $g\ m^{-3}$
Φ_{CBOD} = CBOD concentration, $g\ m^{-3}$
Φ_{sed} = organic sediment concentration, $g\ m^{-3}$
Φ_{macro} = macrophyte concentration, $g\ m^{-3}$
Φ_{zoo} = zooplankton concentration, $g\ m^{-3}$

So why choose mechanistic lake modeling?

- Lake may not be well represented by empirical relationships
- Need to answer more complex management questions
- More robust results(?)



Where are we on the curve?